## [**Chapter 1 Solutions**](http://docs.google.com/chap1.html)

[**Chapter 2 Solutions**](http://docs.google.com/chap2.html)

[**Chapter 3 Solutions**](http://docs.google.com/chap3.html)

[**Chapter 5 Solutions**](http://docs.google.com/chap5.html)

## Exercise 2.19

### Recurrent Relation:

\(T\_0=5 \\ T\_n=nT\_{n-1}+3 \cdot n!,\text{ for }n>0\)

### Apply Summation Factor:

\(a\_nT\_n=b\_nT\_{n-1}+c\_n,\text{ where } a\_n=1,b\_n=n,c\_n=3 \cdot n! \\ s\_na\_nT\_n=s\_nb\_nT\_{n-1}+s\_nc\_n \\ s\_nb\_n=s\_{n-1}a\_{n-1} \\ s\_n=\frac{a\_{n-1}a\_{n-2}...a\_1}{b\_nb\_{n-1}...b\_2} \\ s\_n=\frac{2^{n-1}}{n!} \\ S\_n=s\_na\_nT\_n \\ S\_n=S\_{n-1} + \frac {n! \cdot 3 \cdot 2^{n-1}}{n!}=S\_{n-1}+3 \cdot 2^{n-1}\)

### Unroll The Recurrence:

\(S\_n=S\_{n-2} + 3 \cdot 2^{n-2} + 3 \cdot 2^{n-1} \\ S\_n=S\_0+3\cdot2^0+3\cdot2^1+...+3\cdot2^{n-1} \\ S\_n=S\_0+ \sum\limits\_{k=0}^{n-1} 3 \cdot 2^k\)

### Perturb The Sum:

\(F\_n=\sum\limits\_{k=0}^{n-1} 3 \cdot 2^k \\ F\_n+3\cdot2^n=\sum\limits\_{k=0}^{n} 3 \cdot 2^k \\ F\_n+3\cdot2^n=3\cdot2^0+\sum\limits\_{k=1}^{n} 3 \cdot 2^k \\ F\_n+3\cdot2^n=3+\sum\limits\_{k+1=1}^{n} 3 \cdot 2^{k+1} \\ F\_n+3\cdot2^n=3+\sum\limits\_{k=0}^{n-1} 3 \cdot 2^{k+1} \\ F\_n+3\cdot2^n=3+2\left(\sum\limits\_{k=0}^{n-1} 3 \cdot 2^{k}\right) \\ F\_n+3\cdot2^n=3+2F\_n \\ F\_n=3(2^n-1)\)

### Closed Form Solution:

\(S\_n=S\_0+3(2^n-1)=3(2^n-1)+2\cdot5\cdot\frac{2^{0-1}}{0!}=3(2^n-1)+5 \\ T\_n=\frac{S\_n}{s\_na\_n}=\frac{3(2^n-1)+5}{2\cdot\frac{2^{n-1}}{n!}}= \frac{3\cdot2^n+2}{\frac{2^{n}}{n!}}=\frac{3\cdot n!\cdot2^n+2\cdot n!}{2^n}= 3\cdot n!+\frac{n!}{2^{n-1}}\)

## Exercise 2.21

### Perturb \(S\_n=\sum\limits\_{k=0}^n(-1)^{n-k}\)

\(S\_{n+1}=\sum\limits\_{k=0}^{n+1}(-1)^{n-k+1}= (-1)^0+\sum\limits\_{k=0}^{n}(-1)^{n-k+1}=1-S\_n \\ S\_{n+1}=(-1)^{n+1}+\sum\limits\_{k=1}^{n+1}(-1)^{n-k+1}= (-1)^{n+1}+\sum\limits\_{k=0}^{n}(-1)^{n-k+2}= (-1)^{n+1}+S\_n \\ 1-S\_n=(-1)^{n+1}+S\_n \\ S\_n=\frac{-(-1)^{n+1}+1}2=\frac{(-1)^n+1}2=[n\text{ is even}] \)

### Perturb \(T\_n=\sum\limits\_{k=0}^n(-1)^{n-k}k\)

\(S\_{n+1}=\sum\limits\_{k=0}^{n+1}(-1)^{n-k+1}k= (-1)^0(n+1)+\sum\limits\_{k=0}^{n}(-1)^{n-k+1}k= n+1-T\_n \\ S\_{n+1}=(-1)^{n+1}\cdot0+\sum\limits\_{k=1}^{n+1}(-1)^{n-k+1}k= \sum\limits\_{k=0}^{n}(-1)^{n-k+2}(k+1) \\ S\_{n+1}=\sum\limits\_{k=0}^{n}(-1)^{n-k}k+ (-1)^{n-k}=\sum\limits\_{k=0}^{n}(-1)^{n-k}k+\sum\limits\_{k=0}^{n}(-1)^{n-k} \\ n+1-T\_n=T\_n+S\_n \\ T\_n=\frac{n+1-S\_n}2=(n+[n\text{ is odd}])/2\)

### Perturb \(U\_n=\sum\limits\_{k=0}^n(-1)^{n-k^2}\)

\(S\_{n+1}=\sum\limits\_{k=0}^{n+1}(-1)^{n-k+1}k^2= (n+1)^2+\sum\limits\_{k=0}^{n}(-1)^{n-k+1}k^2= (n+1)^2-U\_n \\ S\_{n+1}=\sum\limits\_{k=1}^{n+1}(-1)^{n-k+1}k^2= \sum\limits\_{k=1}^{n+1}(-1)^{n-k+1}k^2= \sum\limits\_{k=0}^{n}(-1)^{n-k+2}(k+1)^2 \\ S\_{n+1}=\sum\limits\_{k=0}^{n}(-1)^{n-k}(k^2+2k+1)= \left(\sum\limits\_{k=0}^{n}(-1)^{n-k}k^2\right)+2\left(\sum\limits\_{k=0}^{n}(-1)^{n-k}k\right)+\left(\sum\limits\_{k=0}^{n}(-1)^{n-k}\right) \\ (n+1)^2-U\_n=U\_n+2T\_n+S\_n \\ U\_n=\frac{(n+1)^2-2T\_n-S\_n}2=\frac{(n+1)^2-n-[n\text{ is odd}]-[n\text{ is even}]}2=\frac{n^2+2n+1-n-1}2=\frac{n(n+1)}2\)

## Exercise 2.23

### Part A - Summation by Partial Fractions

\(\text{Solve }\sum\limits\_{k=1}^n\frac{2k+1}{k(k+1)} \\ \frac{2k+1}{k(k+1)}=\frac{A}{k}+\frac{B}{k+1}= \frac{A}{k}\cdot\frac{k+1}{k+1}+\frac{B}{k+1}\cdot\frac{k}{k}= \frac{Ak+A+Bk}{k(k+1)}\\ 2k+1=Ak+A+Bk \\ 2(0)+1=A(0)+A+B(0) \\ A=1 \\ 2(1)+1=1(1)+1+B(1) \\ B=1 \\ \sum\limits\_{k=1}^n\frac{2k+1}{k(k+1)}= \sum\limits\_{k=1}^n\frac{1}{k}+\sum\limits\_{k=1}^n\frac{1}{k+1}= H\_n+\sum\limits\_{k=1}^n\frac{1}{k+1}=2H\_n-1+\frac{1}{n+1}=2H\_n-\frac{n}{n+1}\\ \)

### Part B - Summation by Parts

\(\text{Solve }\sum\limits\_{k=1}^n\frac{2k+1}{k(k+1)} \\ \sum u\Delta v=uv-\sum Ev\Delta u \\ u = 2x+1, \Delta v=1/x(x+1)=(x-1)\_{-2} \\ v=-(x-1)\_{-1}=-1/x \\ \Delta u = 2, Ev = -1/(x+1) \\ \sum\frac{(2x+1)}{x(x+1)}\delta x= \frac{-2x-1}{x}+2\left(\sum x\_{-1}\delta x\right)= \frac{-2x-1}{x}+2H\_n \\ \sum\limits\_{k=1}^n\frac{2k+1}{k(k+1)}= \biggl[\frac{-2k-1}{k}+2H\_k\biggr]\_1^{n+1}= \left(\frac{-2n-3}{n+1}+2H\_{n+1}\right)+3-2H\_1 \\ 2H\_{n+1}-\frac{2n+3}{n+1}+\frac{n+1}{n+1}= 2H\_{n+1}-\frac{n+2}{n+1}= 2H\_n+\frac2{n+1}-\frac{n+2}{n+1}=2H\_n-\frac{n}{n+1}\)

## Exercise 2.27

### Compute \(\Delta(c\_x)\)

\(\Delta(c\_x)=c\_{x+1}-c\_x \\ \Delta(c\_x)=c\_x\cdot(c-x-1) \\ \Delta(c\_x)=c\_{x+2}/(c-x) \\ \)

### Deduce \(\sum\limits\_{k=1}^n-2\_k/k\)

\(\Delta(-2\_x)=-2\_{x+2}/(-2-x) \\ \Delta(-2\_{x-2})=-2\_x/-x \\ \Delta(-(-2)\_{x-2})=-2\_x/x \\ \sum\frac{-2\_x}x\delta x=-(-2)\_{x-2} \\ \sum\limits\_{k=1}^n-2\_k/k= \biggl[-(-2)\_{x-2}\biggr]\_1^{n+1}=-(-2)\_{n-1}-1 \\ -2\_{n-1}=-2\cdot-3\cdot...\cdot-n=(-1)^{n-1}n! \\ \sum\limits\_{k=1}^n-2\_k/k=(-1)^nn!-1 \)

## Exercise 2.29

\(\text{Solve }\sum\limits\_{k=1}^n\frac{(-1)^kk}{4k^2-1}\)

### Partial Fractions Expansion

\(\sum\limits\_{k=1}^n(-1)^k\cdot\frac{k}{4k^2-1} \\ \frac{k}{4k^2-1}=\frac{A}{2k+1}+\frac{B}{2k-1}= \frac{2Ak-A+2Bk+B}{4k^2-1}\\ k=2Ak-A+2Bk+B \\ 1=A+3B \\ A=1-3B \\ 2=3A+5B \\ 2=3-9B+5B \\ B=1/4 \\ A=1-3/4=1/4 \\ \frac{k}{4k^2-1}=\frac14\left(\frac{1}{2k+1}+\frac{1}{2k-1}\right)\\ \frac14\sum\limits\_{k=1}^n(-1)^k\left(\frac{1}{2k+1}+\frac{1}{2k-1}\right)\\ \)

### Reduce The Telescoping Sum

\(\frac14\left(-(\frac{1}{3}+\frac{1}{1})+(\frac{1}{5}+\frac{1}{3})-(\frac{1}{7}+\frac{1}{5})+...+(-1)^n(\frac{1}{2n+1}+\frac{1}{2n-1})\right)\\ \frac14\left(-1+(-1)^n(2n-1)\right)=\frac{(-1)^n}{8n-4}-\frac14\)

## Exercise 3.31

\(\zeta(k)=1+\frac1{2^k}+\frac1{3^k}+...=\sum\limits\_{j\ge1}\frac1{j^k} \\ \text{Prove }\sum\_{k\ge2}(\zeta(k)-1)=1 \\ (\sum\limits\_{j\ge1}\frac1{j^k})-1=\sum\limits\_{j\ge2}\frac1{j^k} \\ \sum\limits\_{k\ge2}(\zeta(k)-1)=\sum\limits\_{k,j\ge2}\frac1{j^k}=\sum\limits\_{j\ge2}\sum\limits\_{k\ge2}\frac1{j^k} \\ \sum\limits\_{j\ge2}\sum\limits\_{k+2\ge2}\frac1{j^{k+2}}= \sum\limits\_{j\ge2}\left(\frac1{j^2}\sum\limits\_{k+2\ge2}\frac1{j^{k}}\right)= \sum\limits\_{j\ge2}\left(\frac1{j^2}\sum\limits\_{k\ge0}\frac1{j^{k}}\right) \\ S= \\ S=1+\frac1j+\frac1{j^2}+\frac1{j^3}+...\\ S\cdot j=j+1+\frac1j+\frac1{j^2}+...\\ S\cdot j=j+S \\ S(j-1)=j \\ \sum\limits\_{k\ge0}\frac1{j^{k}}=\frac j{j-1} \\ \sum\limits\_{k\ge2}(\zeta(k)-1)=\sum\limits\_{j\ge2}\left(\frac1{j^2}\cdot\frac j{j-1}\right)=\sum\limits\_{j\ge2}\left(j\cdot\frac 1{j(j-1)}\right) \\ \sum\limits\_{j\ge2}\left(j\cdot\frac 1{j(j-1)}\right)=1+\frac12+\frac13+\frac14+...=1 \)